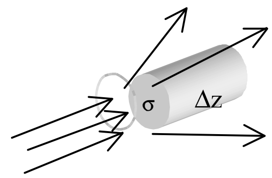
The Energy Loss of Particles in Matter

I. Cross Section

As a particle traverses a matter it has a probability to react by scattering, absorption, or to interaction in the material. The reaction probability is expressed in terms of an energy dependent cross section $\sigma(E)$.



The number of scatters per unit volume n is estimated as the molar mass times Avogadro's #.

$$n = (\frac{\rho}{A})N_A$$

The reaction rate dI/I in a thin slice of material Δz thick and with density ρ is given

$$-\frac{dI}{I} = n\ \sigma(E)\,\Delta z$$

$\sigma(E)$ represents the cross sectional area of a scattering disk of length Δz. The smaller σ, the smaller the probability of reaction (scattering, absorption, interaction) in matter.

The unit of the cross section is the barn, **1 barn = 10^{-24} cm^2.**

Geometric Cross Section

For many applications in low energy scattering the geometrical cross section for atomic or nuclear reactions of can be determined from the atomic or nuclear radius r_A.

$$\sigma_A = \pi\, r_A^2$$

1) What is the neutron nuclear cross section for boron? A =5 $r_A = 1.3x\ 5^{1/3}$F= 2.2F

$$\sigma_A = \pi\, r_A^2 = 15.2\ x\ F^2 = 15.2x10^{-24}\,cm = 15.2\ barns$$

2) When does the approximation break down?

When the deBroglie wavelength of the neutron or incident particle becomes smaller than the nuclear size then the particle will penetrate in to the nucleus and interact with the individual protons and neutron, or quarks.

$$p < h/\lambda \quad \text{where} \quad \lambda \sim r_A$$

II. Interaction of Photons in Matter

Photons interact in matter in three principle ways (1) Photoelectric effect, (2) Compton scattering (3) Pair creation. The total cross section for interaction is given by

$$\sigma(E) = \sigma_{PE} + \sigma_{Com} + \sigma_{Pair}.$$

The photon survival probability in traversing a material of thickness $\boldsymbol{x}$ is expressed as

$$I = I_0 e^{-\mu x} = I_0 e^{-\frac{\rho N}{A}\sigma x}$$

μ is called the absorption coefficient and iis dependent on energy. The absorption coefficient $\mu = \frac{\rho N}{A}\sigma$ is gives the number of scatterers per unit length which the photon encounters.

Half-absorption Length

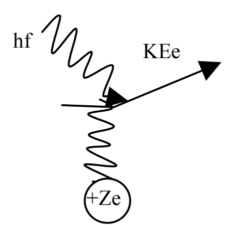
Half the photons are absorbed when I/Io=1/2 = exp($-\mu X_{1/2}$) or $\mathbf{X_{1/2} = 0.693/\mu}$

I. Photoelectric Effect

The general PE effect describes the interaction of a photon of light with an atomic or electron in an energy band of a metal. The kinetic energy of the electron after absorbing photon energy *E=hf* is given by

$$KE_e = hf - \phi$$

where *I* is the ionization potential of the atom or metallic energy band.

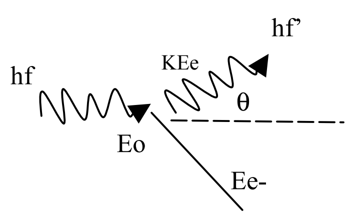


$\sigma_{PE} \sim Z^4 (m_e c^2 / E_\gamma)^3$

Higher *Z* materials are very effective in absorbing photons of energy hf< $m_e c^2$

II. Compton Scattering

In Compton scattering the photon collides with a quasi-free electron in the atom transferring energy to the electron. From energy conservation hf + Eo = hf' + Ee where Eo ~$m_e c^2$.



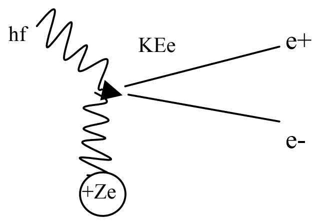
$\sigma_{Com} \sim Z^1 f(E_\gamma)$

The energy photon and electron dependences are given by $E_\gamma' = \dfrac{E_\gamma}{1 + \left(\dfrac{E_\gamma}{m_e c}\right)(1-\cos\theta)}$ *and* $T_e = hf - hf'$

Photon Backscatter (θ=180^o)

A photon of energy $E_\gamma{}' = \frac{E_\gamma}{1+3.91E_\gamma}$ *with* $E(MeV)$ exits the absorber in the direction of the source!

III. Pair Production (hf > 1.022MeV)
In pair production an electron-positron pair is created in the field of a nucleus only if the photon energy exceeds two electron masses, E_γ > 2me. The cross section rises at this threshold and is proportional to Z^2.



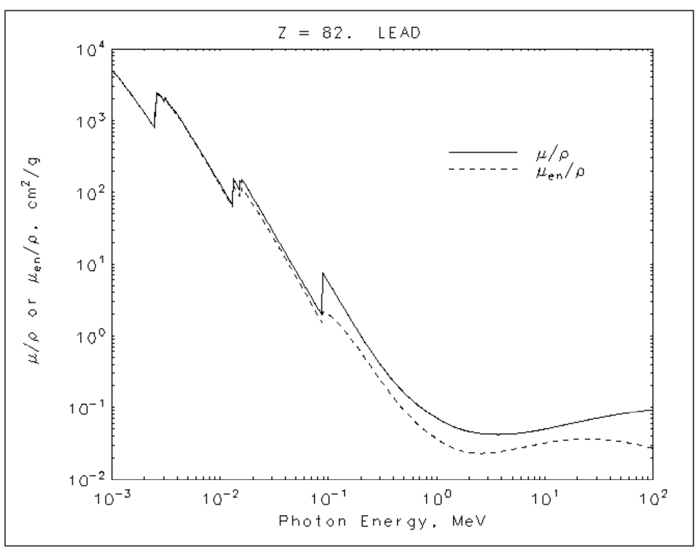
$\sigma_{PAIR} \sim Z^2 g(E_\gamma)$

IV. Mass Absorption Coefficient
Since the cross sections are all dependent on the density of electron scatterers $\rho_\varepsilon \sim \rho$ which is proportional to matter density ρ the aborption coefficient is often replaced by the **mass absorption coefficient**

$$\mu / \rho = \frac{N}{A}\sigma \quad (cm^2 / g)$$

The spiked structure are K-L-M shell electron excitations by the incident photon in lead.

http://physics.nist.gov/PhysRefData/XrayMassCoef/tab3.html



Mass absorption coefficient in Lead.

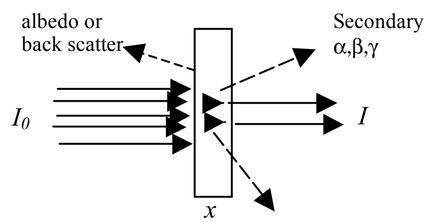
At about 1 MeV photons are not highly absorbed even in Lead!

III. Attenuation Length of a Photon Beam in a thin Absorber

A beam of photons will be attenuated in the forward direction in a material of thickness *x* based on simple scattering or absorption processes. The linear attenuation coefficient μ scales with material density ρ, and the **mass attenuation coefficient** μ/ρ *is* often reported.

$$-\frac{dI}{I} = (\frac{\rho N_A}{A})\sigma \ dx = \mu \ dx$$

$$I = I_0 \ e^{-\mu X}$$



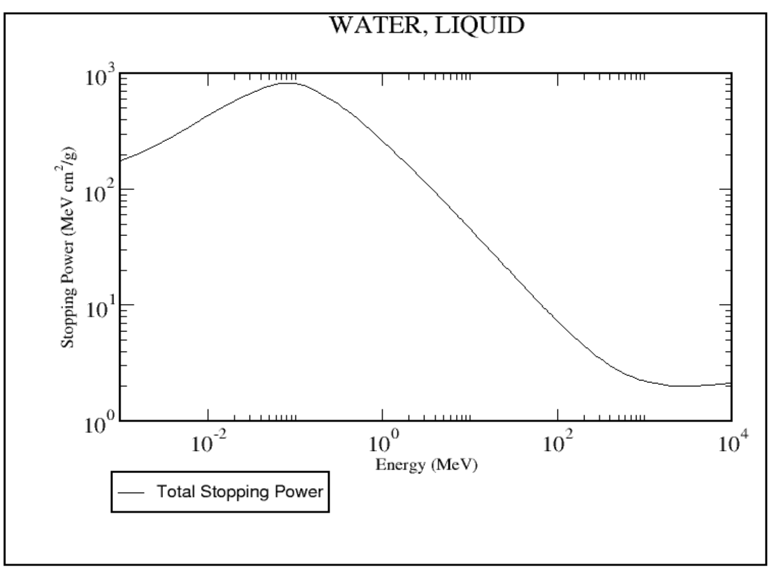
Part of the beam may be absorbed in the material or scattered out of the beam. Sometimes the beam may create secondary scattering processes and new forms of α, β ,γ radiation may emerge from the absorber.

IV. Energy loss by Heavy Charged Particle (Stopping Power)

Heavy charged particles (charge *z*) loose energy principally be ionization processes in materials. The ionization cross section is denoted by $\sigma_{ionization}$ and measured in barnes (10^{-24} cm^2) The Bethe Bloch formula gives the energy dependence of the energy loss. The electron density $n_e = \overbrace{\frac{\rho N_A}{A}}^{atoms/cm^3} \cdot Z$

Alpha particles (z=2) will loose four times as much energy in matter as protons (z=1).

$$-\frac{dE}{dx} = \frac{4\pi z^2}{mc^2\beta^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 n_e \cdot \ln\left(\frac{2mc^2\beta^2}{I(1-\beta^2)}\right) - \beta^2 \quad Bethe-Bloch \ Formula$$

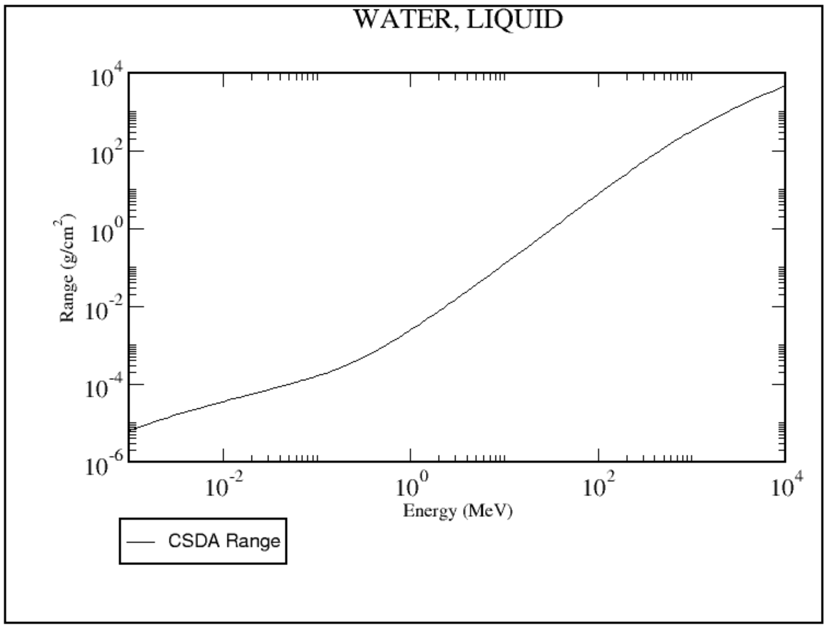


http://pdg.lbl.gov/2009/AtomicNuclearProperties/index.html

V. Range

We can determine the mean range of of heavy charged particles of kinetic energy *T* by integration of the stopping power *-dE/dx*.

$$R(T) = \int_0^T \left(\frac{dE}{dx} \right)^{-1} dE$$



VI. Range of Alpha Particles in Air

The range of alpha particles in air can be parameterized as

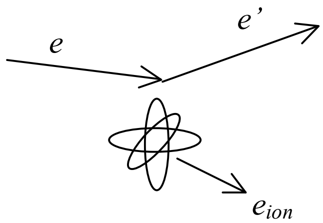
R(cm) = 0.4 E(MeV) $E\alpha < 4MeV$
R(cm)= 1.24E – 2.62(MeV) $4 < E\alpha < 8MeV$

VII. Electron Energy loss in Matter by Ionization and Bremsstrahlung

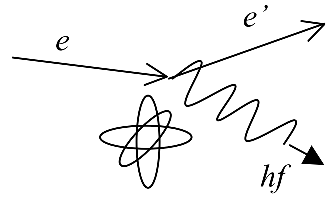
Electrons (*z*=1) lose energy by (a) ionization processes similar to Bethe-Bloch formula for heavy charged particles. In addition electrons lose energy by (b) radiation or Bremsstrahlung (breaking radiation). Bremsstrahlung radiation is negligible for protons and alphas particles.

$$\left(-\frac{dE}{dx}\right)_{ion} = \frac{4\pi z^2}{mc^2\beta^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 n_e \cdot \ln\left(\frac{mc^2\tau\sqrt{\tau+2}}{\sqrt{2}I}\right) - F(\beta) \qquad \tau = T_e / mc^2 \qquad (a)$$

$$\left(-\frac{dE}{dx}\right)_{rad} \simeq \frac{Z \cdot E(MeV)}{800}\left(-\frac{dE}{dx}\right)_{ion} \qquad (b)$$



Ionization

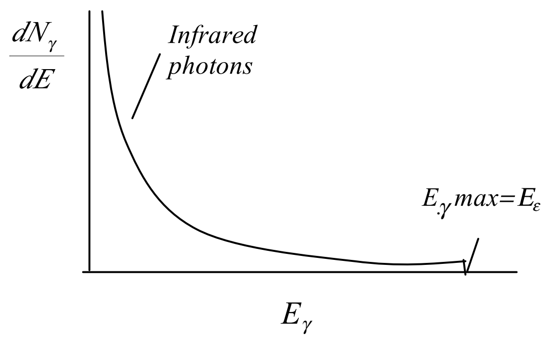


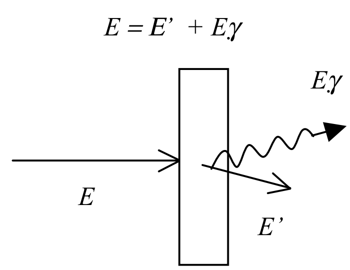
Radiation

The radiation yield *Y*, is the fraction of the total *dE/dx* of electron of kinetic energy *T*, producing radiative photons. *T* in MeV.

$$Y = \frac{\left(-dE/dx\right)_{rad}}{\left(-dE/dx\right)_{rad} + \left(-dE/dx\right)_{ion}} \simeq \frac{6\times 10^{-4} Z\ T}{1+6\times 10^{-4} Z\ T} \equiv \text{ Radiation Yield of stopped electron}$$

The **radiation yield** of an absorber can be a dangerous source of secondary radiation! The number of gamma rays produced falls as $1/E\gamma$ but higher energy photons at the end of the spectrum will escape the absorber. The radiation flows in the direction of the incident electron beam with a $\Delta\theta \sim 1/E_\gamma$ distribution .



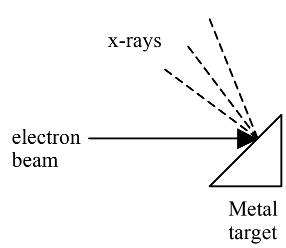


Va. X-rays production by electrons

X-rays are copiously produced by stopping electrons in a heavy metal target of high-*Z* material. The generated x-ray spectra is composed of a (a) continuous bremsstrahlung portion and (b) an x-ray line spectra described by the Bohr model. *Zeff* takes in to account the effective shielding of the nucleus by inner oribital electrons and other spin-orbit and spin-spin effects.

$$\left(\frac{d\sigma}{dk}\right)_{Brem} \sim \left(\frac{Z^2}{T_i}\right)\frac{1}{k}\ln\left(\frac{4T_i}{k}\right) \qquad (a)$$

$$E_{L\alpha} \simeq Z_{eff}^{\ 2} \times 13.6eV \ \left(\frac{1}{n_1^{\ 2}} - \frac{1}{n_2^{\ 2}}\right) \qquad (b)$$



In a typical electron x-ray tube, electrons are accelerated to the target by Vacc = 10's of KVolts of potential. A 100KV x-ray tube will produce at best 10 KeV x-rays !

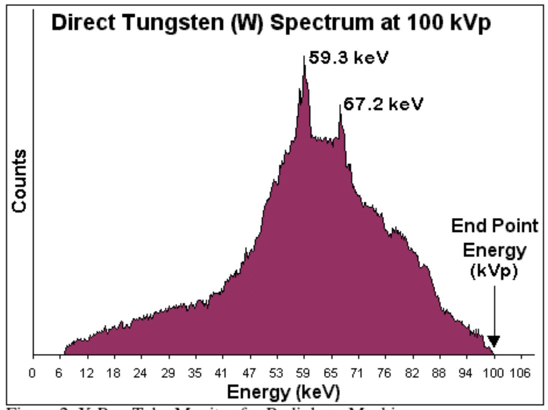


Figure 2. X-Ray Tube Monitor for Radiology Machines

The bremsstrahlung is caused by the de-acceleration of the electron in the metal. The x-ray lines are from the filling of inner core electrons that have been displaced.

K,L,M x-rays are produced in materials when inner atomic electrons are dislodged or ionized. Following the Bohr model, Mosely predicted x-rays would follow the pattern:

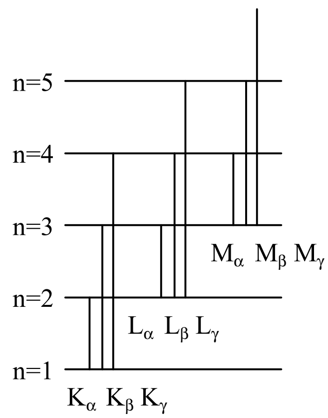
$$E_{K\alpha} \simeq (Z-1)^2 \times 13.6eV \ (\frac{1}{1^2} - \frac{1}{2^2})$$

$$E_{K\beta} \simeq (Z-6)^2 \times 13.6eV \ (\frac{1}{1^2} - \frac{1}{3^2})$$

$$E_{L\alpha} \simeq (Z-6)^2 \times 13.6eV \ (\frac{1}{2^2} - \frac{1}{3^2})$$

$$E_{L\beta} \simeq (Z-6)^2 \times 13.6eV \ (\frac{1}{2^2} - \frac{1}{4^2})$$

etc.



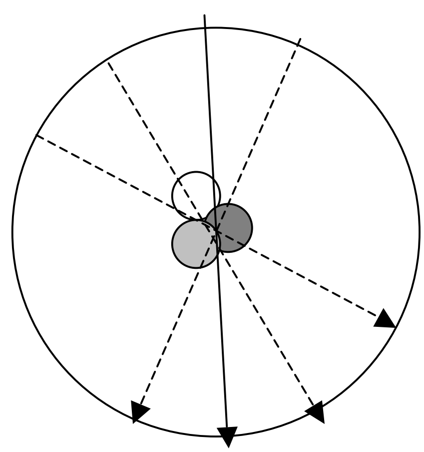
Computed Tomography

The attenuation of X-rays passing through an object can be used for imaging – computed tomography. For a monoenergetic x-ray beam and a variable density absorber we can integrate over the line of sight of the beam and obtain the image after many measurements are made. A typical scenario of a CT scan is shown below.

$$I_{x,y} = I_0 \ e^{-\int \mu(x,y)\, s}$$

$$\ln(I_0 / I_{x,y}) = \int \mu(x,y) ds$$

$$\mu(x,y) = \left[\ln(I_0 / I_{x,y})\right]^{-1} \equiv \text{x-ray image}$$



Questions to Answer:

1) Estimate the kinetic energy at which collisional and radiative stopping power are equal for electrons in (a) Be , (b) Cu, (c) Pb.

2) Estimate the fraction of the energy of a 2 MeV beta ray that is converted in to bremsstrahlung when the particle is absorbed in lead.

3) Use Table 6.1 to estimate the range in cm in air for electrons of (a) 50keV, (b) 830 keV, (c) 100 MeV

4) Use Table 5.3 to determine the minimum energy that a proton must have to penetrate 30cm of tissue, the approximate thickness of the human body.

5) Using table 5.3 for protons mass stopping power of water, estimate the stopping power of Lucite (ρ=1.19g/cc) for a 35 MeV proton.

6) How far will 10 MeV alphas penetrate in air (ρ=1.24 x 10^{-3}g/cc, STP)

7) Estimate the energy of the Kα X-ray in Pb.

TABLE 6.1. Electron Collisonal, Radiative, and Total Mass Stopping Powers; Radiation Yield; and Range in Water

Kinetic Energy	β^2	$-\frac{1}{\rho}\left(\frac{dE}{dx}\right)_{col}$ (MeV cm² g⁻¹)	$-\frac{1}{\rho}\left(\frac{dE}{dx}\right)_{rad}$ (MeV cm² g⁻¹)	$-\frac{1}{\rho}\left(\frac{dE}{dx}\right)_{tot}$ (MeV cm² g⁻¹)	Radiation Yield	Range (g cm⁻²)
10 eV	0.00004	4.0	—	4.0	—	4×10^{-8}
30	0.00012	44.	—	44.	—	2×10^{-7}
50	0.00020	170.	—	170.	—	3×10^{-7}
75	0.00029	272.	—	272.	—	4×10^{-7}
100	0.00039	314.	—	314.	—	5×10^{-7}
200	0.00078	298.	—	298.	—	8×10^{-7}
500 eV	0.00195	194.	—	194.	—	2×10^{-6}
1 keV	0.00390	126.	—	126.	—	5×10^{-6}
2	0.00778	77.5	—	77.5	—	2×10^{-5}
5	0.0193	42.6	—	42.6	—	8×10^{-5}
10	0.0380	23.2	—	23.2	0.0001	0.0002
25	0.0911	11.4	—	11.4	0.0002	0.0012
50	0.170	6.75	—	6.75	0.0004	0.0042
75	0.239	5.08	—	5.08	0.0006	0.0086
100	0.301	4.20	—	4.20	0.0007	0.0140
200	0.483	2.84	0.006	2.85	0.0012	0.0440
500	0.745	2.06	0.010	2.07	0.0026	0.174
700 keV	0.822	1.94	0.013	1.95	0.0036	0.275
1 MeV	0.886	1.87	0.017	1.89	0.0049	0.430
4	0.987	1.91	0.065	1.98	0.0168	2.00
7	0.991	1.93	0.084	2.02	0.0208	2.50
10	0.998	2.00	0.183	2.18	0.0416	4.88
100	0.999+	2.20	2.40	4.60	0.317	32.5
1000 MeV	0.999+	2.40	26.3	28.7	0.774	101.

TABLE 5.3. Mass Stopping Power $-dE/\rho dx$ and Range R_p for Protons in Water

Kinetic Energy (MeV)	β^2	$-dE/\rho dx$ (MeV cm² g⁻¹)	R_p (g cm⁻²
0.01	.000021	500.	$3 \times 10^{-}$
0.04	.000085	860.	$6 \times 10^{-}$
0.05	.000107	910.	$7 \times 10^{-}$
0.08	.000171	920.	$9 \times 10^{-}$
0.10	.000213	910.	1×10
0.50	.001065	428.	$8 \times 10^{-}$
1.00	.002129	270.	0.002
2.00	.004252	162.	0.007
4.00	.008476	95.4	0.023
6.00	.01267	69.3	0.047
8.00	.01685	55.0	0.079
10.0	.02099	45.9	0.118
12.0	.02511	39.5	0.168
14.0	.02920	34.9	0.217
16.0	.03327	31.3	0.280
18.0	.03731	28.5	0.342
20.0	.04133	26.1	0.418
25.0	.05126	21.8	0.623
30.0	.06104	18.7	0.864
35.0	.07066	16.5	1.14
40.0	.08014	14.9	1.46
45.0	.08948	13.5	1.80
50.0	.09867	12.4	2.18
60.0	.1166	10.8	3.03
70.0	.1341	9.55	4.00
80.0	.1510	8.62	5.08
90.0	.1675	7.88	6.27
100.	.1834	7.28	7.57
150.	.2568	5.44	15.5
200.	.3207	4.49	25.5
300.	.4260	3.52	50.6
400.	.5086	3.02	80.9
500.	.5746	2.74	115.
600.	.6281	2.55	152.
700.	.6721	2.42	192.
800.	.7088	2.33	234.
900.	.7396	2.26	277.
1000.	.7658	2.21	321.
2000.	.8981	2.05	795.
4000.	.9639	2.09	1780.